The Cremonese System for Positioning the F-Holes ©

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Of all the "secrets" of Stradivari, the one I find most tantalizing is the matter of positioning the f-holes. Minute variations may not have major acoustical effects: one can easily find wonderful instruments where the distances between upper eyes vary from 36 mm to well over 44. Nor are perfect design, placement, and slant esthetic mandates: connoisseurs have extolled the beauty of soundholes that are upright, slanted, high, low, cockeyed, off-center, and highly asymmetrical. Rather, this secret tantalizes in the way that a pirate's treasure map tantalizes when it lacks distances, reference points, and orientation. Without some missing key you cannot use it yourself.

Stradivari's soundhole diagrams (Figure 1) show that the master carefully worked out the positioning of the holes. Intersecting arcs precisely place the centers of the eyes for drilling, so the diagrams are operationally useful. But why are the arcs centered where they are? Why are they the radius shown and not something different? Without answers, makers today who want to remain within the Cremonese tradition but develop their own models have no guide to placing soundholes.

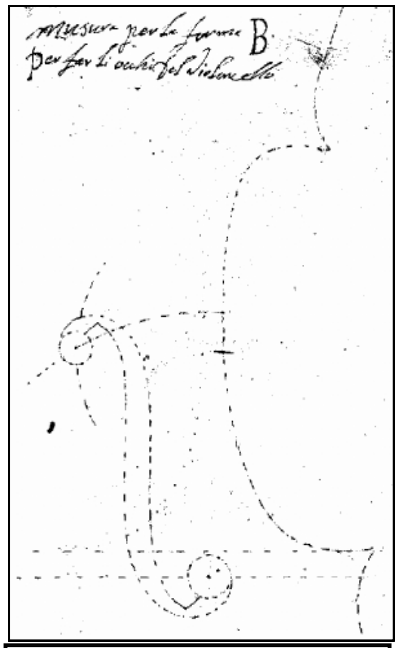


Figure 1- Stradivari's Diagram for the f-holes on the Form B Cello

Principles for a System

Here are some criteria for any plausible system: the system

- should be simple, requiring nothing more than a ruler and divider
- should precisely locate both the upper and lower eyes so they can be drilled accurately
- should be consistent with the paper diagrams of Stradivari
- should explain and be consistent with any known marks used to locate the f-holes that remain visible on the instrument
- should produce the "errors" in soundhole placements that are commonly observed: f-holes are higher on one side than on the other; f-holes are off-center; f-holes have different slants; f-holes are different distances from the edges, etc.
- should produce the observable consistencies; chief among these is that the distances between the centers of the upper and lower eyes are usually the same on treble and bass.

* I am grateful to Andrew Dipper, Robert Spear, and especially Gary Frisch for questions and suggestions that greatly improved this paper.

The Cremonese System[1]

The Golden Section

My research suggests that the Cremonese system for setting the size and position of the f-holes had two elements. The first is a concept known as the "Golden Section" (GS) of a line. The GS—sometimes called the "divine proportion"— divides a line into two parts such that the ratio of the whole line to the longer part is the same as the ratio of the longer part to the smaller part. Algebraically, given a line of length AB, with a subdivision at point C, point C will be located at the Golden Section of the line AB if AB is to AC as AC is to CB.

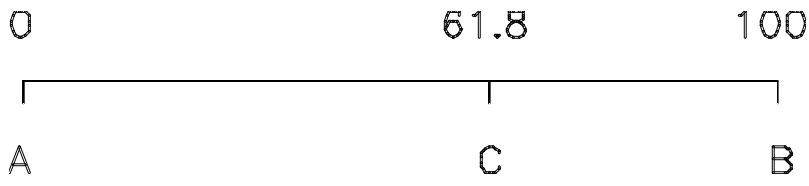


Figure 2 – The Golden Section of a Line

The Cremonese makers usually placed the middle of the violin's lower f-hole eyes on the Golden Section of the length between the upper and lower pins. They used pin length, not instrument length from edge to edge, to establish basic measurements for the f-holes.[2] Makers used the pin length, irrespective of whether the pins are close to the edge—indeed, bisected by the purfling as with Stradivari—or well inside the edge.

On the larger instruments, particularly the tenor violas and cellos, makers used the GS of the instrument pin length to position the lower f-hole eye. But, probably to shorten the stop length and make the instrument easier to play, they altered the placement of the GS and the relationship of the f-hole to the GS. For example, on his violas, Stradivari found the GS of the pin length, but when laying out the f-holes, he measured from the top edge rather than from the upper pin. On his tenor violas and Form B cellos, Stradivari positioned the bottom of the f-hole at the GS. These changes and others shorten the Form B cello's f-holes by about 10 mm from what they would be if the instrument were proportioned like a violin.

The Cremonese makers incorporated the GS into their violin designs at a second stage in some instruments. Often, lines drawn through the centers of the upper and lower eyes will cross, when extended, at a point that is itself a Golden Section of the distance between the upper pin and the line on which the lower eyes lie (I call this 'GSGS'). (Figure 3)

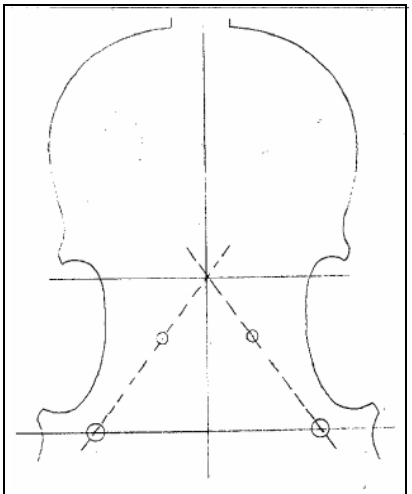


Figure 3 The "Cremonese" Stradivari of 1715 showing the positions of the eyes and the two Golden Sections.

[1] For complete details see my "The Cremonese System for Positioning f-holes: a Plausible Re-creation"

[2] The few exceptions I have found are some very short instruments (less than about 352 mm) of the Guarneri family that use the total length.

The Proportioning Triangle

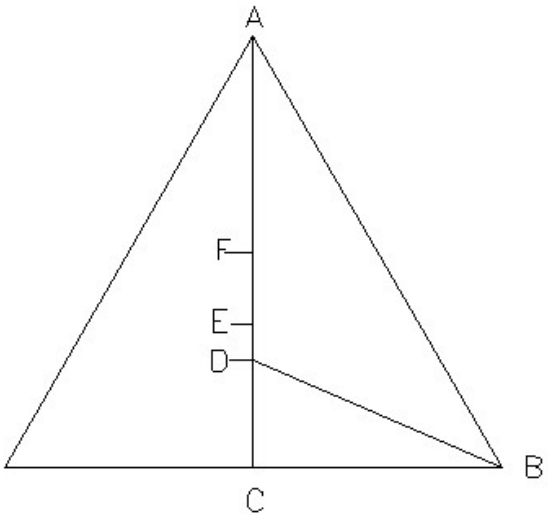


Figure 4 – The Proportioning Triangle: The distance AB is one-third the pin length. BC is one-half the length of the side; therefore, one-half AB. The line AC is perpendicular to the side opposite point A. CD is one-fourth AC. CE is one-third AC. CF is one-half AC. BD is the hypotenuse of the right triangle BCD.

The second element of the Cremonese design system for f-holes is an equilateral triangle including various sub-divisions as shown in Figure 4. The side of the triangle, AB, is one-third the pin length. Rarely, the side of the triangle is one-third the total length of the instrument.[3]

For the usual Cremonese instrument—Amati family, Guarneri family, Stradivari, Bergonzi, Rogeri, Ruggieri—the critical measurements for the size and orientation of the f-holes will all be drawn from the distances AB, BC, AC, CD, CE, CF, and BD, where AB is one-third the pin length of the instrument.

The Elements Combined

Cremonese craftsmen would have needed a way to partition a line at the Golden Section using only dividers and a straightedge. Remarkably, the side of the equilateral triangle based on the instrument's pin length and the perpendicular of that triangle when added together equal the Golden Section of the pin length! Consequently, there is a quick and simple way for the craftsman to find the Golden Section [Figure 5]:

- Denote the ends of a line as U and L
- Use a divider to find the points (B and K) that divide the line into three parts
- With the divider set at one-third the total length, construct the equilateral triangle having the base UB
- From the apex A, drop a perpendicular to UB (easily done because the perpendicular will bisect UB). Label the intersection C.
- With the divider set at distance AC, draw a small arc intersecting line UL from point B

[3] To my knowledge, Andrew Dipper, in his booklet, Musical Instruments of the Violin Family and the Geometry of Positioning and Cutting Out Their F-Shaped Sound Holes, was the first to observe that often one can locate the position of the lower eyes of an Amati-family violin using an equilateral triangle having sides one-third the instrument's pin length. If the apex of the triangle is placed on the centerline, one-third down from the upper pin, the center of the lower eyes lies on or very close to the lower corners of the triangle.

- The intersection is the Golden Section of the distance UL.

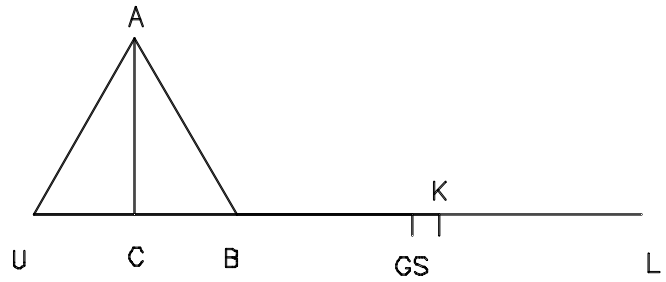


Figure 5 – The Link Between the Golden Section and the Proportioning Triangle—The Golden Section is the geometric sum of the distance AB and AC

Applied to the violin, the upper and lower pins define the length UL. One-third the pin length defines a triangle having sides AB, from which it is easy to find the length AC. AB plus AC yields the Golden Section for placing the lower eyes. The distance BD in Figure 4, or sometimes BC, sets the length of the f-holes, as measured by the distance between the upper and lower eyes. Various subdivisions of AC fix the position with respect to the edges or centerline. Thus, from one measure, the distance between the pins, the Cremonese maker could readily determine the exact size and position of the f-holes, quickly and easily, using no more than a divider and straightedge.

Interestingly, this system does not apply to makers outside the Cremonese tradition.

The geometrical construction just described provides a close but not perfect approximation to a Golden Section. Mathematically, the Golden Section is an irrational number starting out 0.618….. The geometrical approximation is slightly greater, at 0.622. It is significant that the system for sizing and positioning the f-hole on Cremonese instruments works better when using the geometrical approximation than when using the accurate value, suggesting to me that this is indeed what the Cremonese craftsmen did. The important exception to this is Stradivari as I will explain next month.

Three Applications

Andrea Amati

Consider the Andrea Amati violin of 1564 (The Strad, December 1991). The straightline distance between the pins on the poster is 325 mm. One third this length is 108.3 mm. Construct the equilateral triangle with sides, AB, of 108.3 mm. A little algebra will show that for such a triangle AC is 93.8 mm. CD is 23.5 mm and BD is 59.0 mm[4]. In the Cremonese

[4] The first seven equations derive the length of the perpendicular as a function of the side of the equilateral triangle, using the usual Pythagorean theorem for right triangles. The second set of equations derives the length BD as a function of the distance AB. These calculations are useful for the researcher, but are absolutely not needed by the craftsman who would use only a divider and straightedge

system, the length of the violin fhole is BD. Very rarely on violins, but almost always on tenors and cellos, the length of the f-hole is BC. Note that by "length," I mean the distance between the centers of the upper and lower eyes. Regrettably, instrument documentation almost never provides this measure, giving instead the distance between the highest and lowest points on the wings. That distance is of aesthetic interest, but it is highly variable and does not properly describe what the maker considered in placing the f-hole.

I measure the f-hole length on the poster at 58.5 mm, which compares well with the calculated length of 59, bearing in mind the many possible sources of distortion and error.

Lower eyes

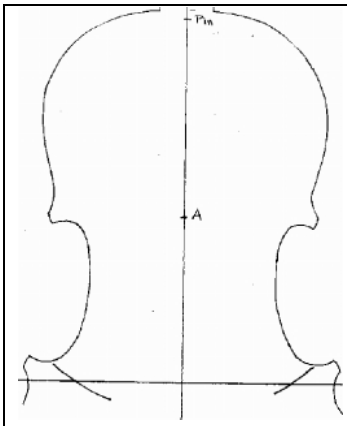


Figure 6 - Positioning the Lower Eyes Using arc AB and the Golden Section of the Pin Length

A variety of geometric procedures, usable at the workbench, will place the lower eyes properly. The simplest sets the lower eye at the intersection of an arc of radius AB, positioned AB down from the upper pin and a line perpendicular to the centerline at the Golden Section. Operationally, the maker uses a divider to find the distance AB that is one third the pin length. With the divider opened to AB, find point A on the centerline, which is one third the pin length or AB below the upper pin. From that point draw two short arcs in the vicinity of the lower eyes. Then, resetting the divider to the length AC and positioning it again at point A, mark the intersection with the centerline and draw a perpendicular. That establishes the Golden Section. Where the perpendicular to the centerline at the GS intersects the two arcs previously drawn mark the center of the lower eyes. [Figure 6]

$$AB^2 = AC^2 + CB^2$$
$$AC^2 = AB^2 - CB^2$$
$$AC^2 = AB^2 - (1/2\,AB)^2$$
$$AC^2 = AB^2 - 1/4\,AB^2$$
$$AC^2 = 3/4\,AB^2$$
$$AC = \sqrt{3/4}\,AB$$
$$AC = .8660254\quad AB$$

$$BD^2 = (1/4\,AC)^2 + (1/2\,AB)^2$$
$$BD^2 = (1/4)^2 * 3/4\,AB^2 + 1/4\,AB^2$$
$$BD = \sqrt{19/64}\,AB^2$$
$$BD = .5448623\quad AB$$

Upper eyes

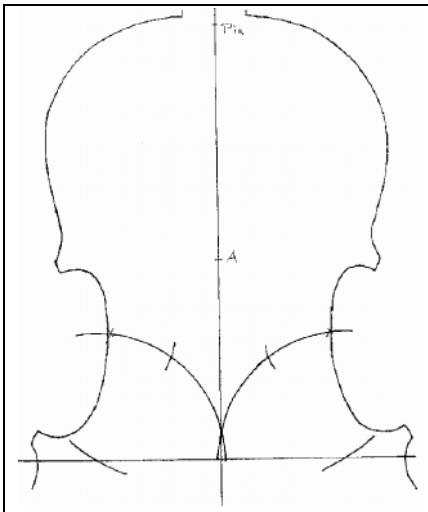


Figure 7 – Positioning the Upper Eyes from the Centers of the Lower Eyes and the Edges of the C-bouts

The upper eyes are positioned with reference either to the centerline or to the edge of the plate in the middle part of the C-bouts. It is usually easy to tell which system the maker followed because the eyes will be symmetrical to one marker but not the other. The upper eyes of the 1564 Amati are located relative to the edges: open the divider to length BD, and mark the arc centered on the lower eye. Reset the divider to length CD (1/4 AC). Place one point of the divider where the arc BD intersects the rib scribe line, and draw a short arc intersecting the arc BD. The upper eye will lie on this point (Figure 7).

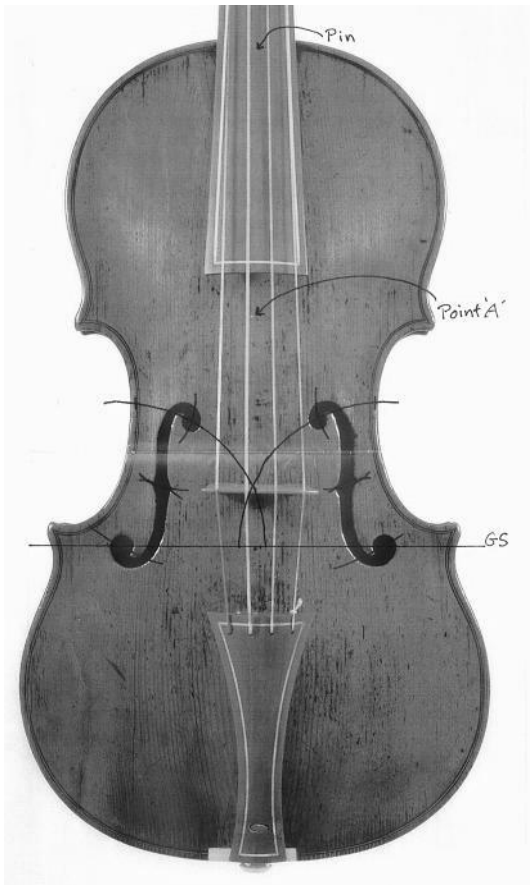


Figure 9 - A. Amati 1564 showing placement of f-holes

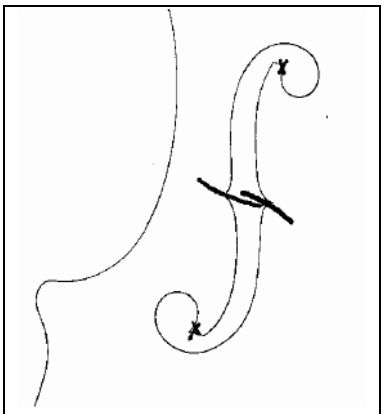


Figure 8 – Placing the notches at 1/3 AC from the inner points of the wings on 1564 Amati

At this point, the eyes can be drilled and the body of the f-hole can be positioned using the template, marked, and cut.

f-hole notches

The notches on the f-hole are located by setting the divider at length CE (1/3 AC). Place one point of the divider on the corner of the wing closest to the center of the eye and mark the intersection with the edge of the f-hole (Figure 8). For Amati instruments the arc from the divider on the lower wing often marks the position on the inner edge of the f-hole, while the arc from the divider on the upper wing marks the position on the outer edge of the f-hole.

Nicolo Amati

One hundred years later Nicolo Amati used almost the same system as his grandfather to fix the size and position of the f-holes on his violin of 1649.

Lower eyes: the lower eyes are placed exactly as for the 1564 Andrea Amati.

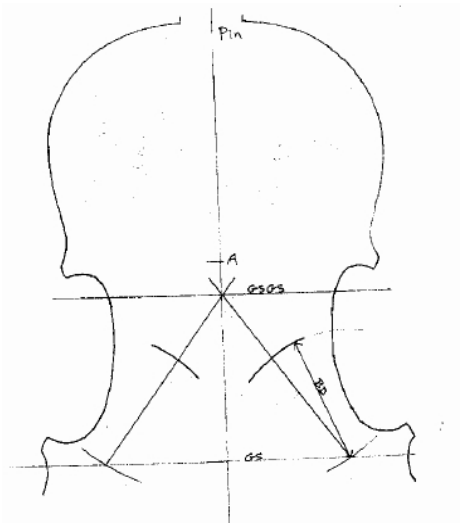


Figure 10– Using BD and GSGS to Place Upper Eyes

Upper eyes: unlike the grandfather's instrument, lines through the upper and lower eyes of the Nicolo Amati violin cross at the GSGS. For such instruments, the maker can position the upper eyes as follows: use dividers to mark the GSGS at the centerline; draw a line between the middle of the lower eye and GSGS; open the divider to radius BD and draw the arc centered on the lower eye; set the upper eye where the arc intersects the line (Figure 10).

Alternatively, the upper eye can be placed without explicit reference to the GSGS as follows: open the divider to BD and draw the arc centered on the lower eye (Figure 11). On the centerline, locate the point that is 1/4 AC above the line through the lower eyes. Open the divider to 1/3 AC, place one point at 1/4 AC and mark two arcs intersecting with the arcs BD. The upper eyes will lie at the intersections. On this highly symmetrical instrument, the upper eyes can be located in yet another way: Open the divider to 1/3 AC and place one point where arc BD intersects the edge of the plate. Mark two arcs intersecting the arcs BD.

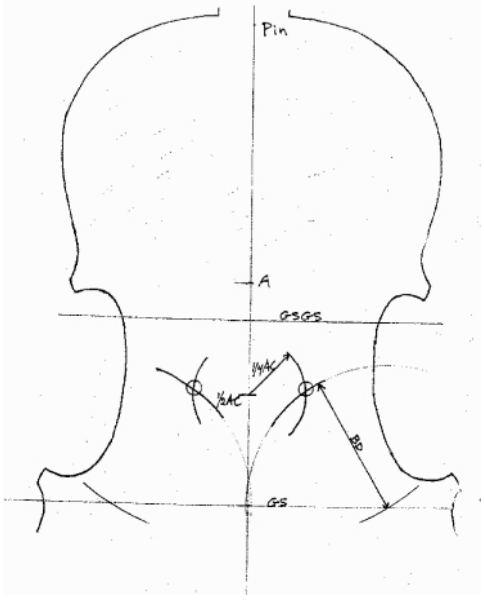
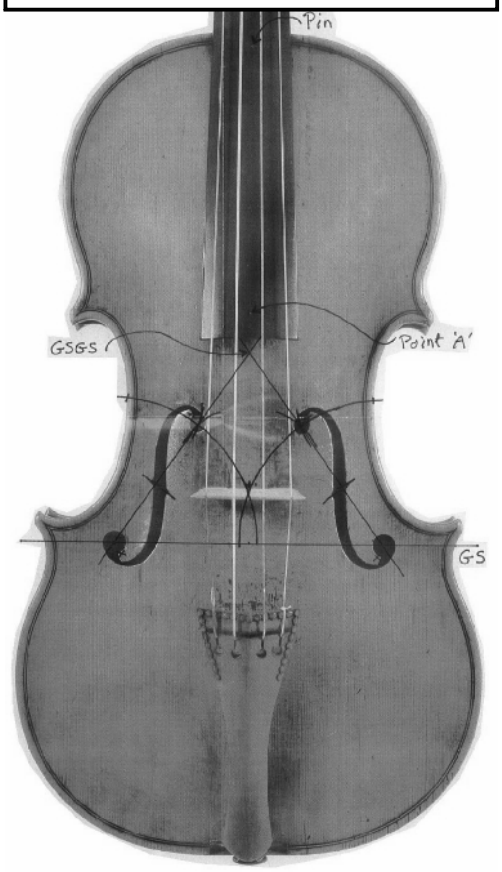


Figure 11 – Using One-half and One-quarter AC to Place Upper Eyes on Arc BD

Figure 12 - 1649 N. Amati showing placement of f-holes



Either of the first two alternatives places the upper eyes relative to the instrument's centerline. The system that locates the upper eyes relative to the centerline by placing a divider at a point along AC (at ¼ or ½ AC) and inscribing an arc that is some fraction of AC is quite common.

f-hole Notches: the notches are placed by arcs of 1/3 AC centered on the inner points of the wings

Giuseppe Guarneri del Gesu

Most Strad readers are familiar with the wildly unconventional shapes of the soundholes on del Gesu's violins. In their book on the Guarneri family, the Hills stated that

> …it is apparent to the experienced observer that del Gesu both placed and cut [the soundholes]… without any close adherence to fixed principle or pattern. He simply relied upon a rough-and-ready determination of the position of top and bottom holes, and the rest depended upon the fancy of the moment.[5]

I find, however, that although the shapes of the soundholes may be odd, the eyes are carefully, even meticulously, placed exactly as prescribed by the system applicable to the earliest Amati. An important fact is that several of del Gesu's violins have clear traces of the arc BD scratched across the upper wing of the f-hole. Violins where the scratch is visible include the Plowden (both upper wings), Stern (treble), Kemp (both), Kortshak (treble), Ysaÿe (bass), Vieuxtemps (treble), and Sauret (bass). These scratches support the use of the system I have described and also indicate that del Gesu positioned the f-holes working from the outside of the instrument.

The 'Alard' of 1742

The distance between the pins on the back is 341.6 mm[6], giving an AB of 113.9 and GS of 212.5. AC is 98.6 and BD is 62.1.

Lower eyes: use the divider to locate point A at 113.9 mm down from the upper pin. Note that on this violin, point A is placed off the centerline at about 2mm toward the treble side. This will have important consequences. From point A, use the divider to mark the arcs AB in the vicinity of the lower eyes. Then, starting from just above the upper pin, measure the GS as AB plus AC. At the GS, draw the perpendicular; the intersections with the arcs set the centers of the lower eyes.

Upper eyes: open the dividers to ½ AC and put one leg on A (on the centerline this time) and mark arcs in the vicinity of the upper eyes. Reset the divider to 1/3 AC, put one leg on the centerline at the GS, and find the point 1/3 AC up the centerline. From that point, mark arcs of 1/3 AC to intersect with the arcs from point A. The intersections mark the centers of the upper eyes.

f-hole Notches: set the divider at 1/3 AC and position one leg on the arc AB that set the lower eyes, roughly in the middle of the lower wing. The notches are at the intersections of the arcs and the edges of the f-holes.

This violin shows several variants of the basic system. It also shows how one small error in layout translates into f-holes that are distinctly off-center (the lower eyes are closer to the treble edge) and of different lengths (the bass f-hole is 62.8 mm but the treble is 64.6; neither matches BD). Because point A for locating the lower eyes was set off-center toward the treble, the intersections of the arcs AB and the GS are also shifted toward the treble edge.

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[5] W. H. Hill, A. F. Hill, A. E. Hill, The Violin-Makers of the Guarneri Family (1626-1762), p.82.

[6] The measurement on the photo is 343 mm, but the photo is slightly oversized relative to the stated dimension and had to be scaled.

Normally, such an error would not cause a difference in the lengths of the f-holes on the treble and bass because both upper eyes would be set on arcs BD from the lower eyes. But on this instrument the upper eyes are set from point A on the centerline and with no reference to BD. The combination of placing point A for the lower eyes off-center and positioning the arcs for setting the upper eyes on-center perfectly explains the differences in the f-hole lengths.

There are two other points of interest. First, the GS length is based on pin length but is measured from just above the upper pin. The effect is to increase the width between the lower eyes, making them 123.9 rather than 113.9 mm as they would have been on an Amati of the same pin length. Second, it is unusual to position the upper eyes of a violin using two arcs of 1/3 AC; it is more usual to go up ¼ AC from the GS line and then use an arc of 1/3 AC to set the centers of the eyes. The effect here is to widen the distance between the upper eyes to 43.5 mm edge–to-edge. Perhaps del Gesu did this to be consistent with the greater width between the lower eyes.

Figure 13 – 'Alard' del Gesu showing placement of f-holes

The Cremonese System

The Cremonese system for determining the size and placement of the f-holes turns out to be just as one would expect: simple in concept, easy to implement using only the most basic

tools, and extremely flexible. Using different combinations and minor variations of the basic rules, the Cremonese makers could have a personal look with as much consistency or inconsistency as they chose while retaining a distinctive "Cremonese" appearance to the instruments. Using the system I have set out, a maker today can create his or her own personal design while retaining the look of the old Italian instruments.

Next month Alvin Thomas King tells how Stradivari modified the Amati system and explains the puzzling diagrams for positioning the fholes.