How Stradivari Positioned the F-Holes ©[**]

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To position the f-holes on his early instruments Stradivari used without significant change the system Andrea Amati developed. Later, Stradivari made a small change that ultimately had big consequences in layout and especially in workshop practices. When I introduced the concept of the Golden Section (GS) of a line last month, I noted that the sum of AB and AC in Figure 1 is only a close approximation to the true Golden Section (0.622 versus 0.618). For Amati and Guarneri family instruments the approximation actually provides a better match to the size and position of the f-holes than does the accurate value. Through the early 1690s the approximation fits Stradivari's instruments also. It seems that Stradivari found a new, more accurate, geometric process in the mid-1690s and used it thereafter.[1]

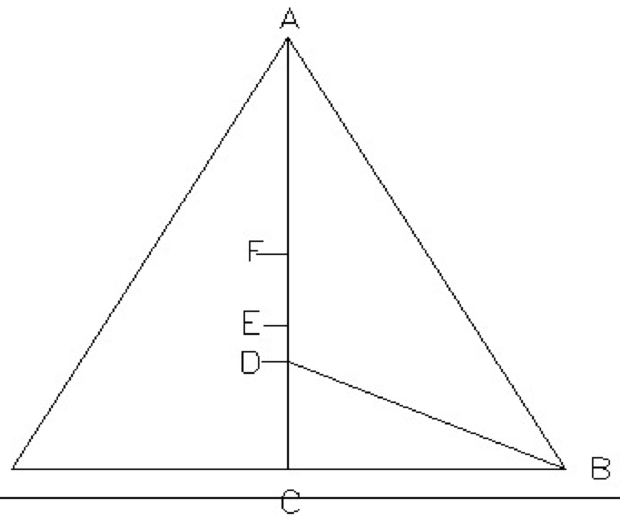

Figure 1 – The Proportioning Triangle: The distance AB is one-third the pin length. BC is one-half the length of the side; therefore, one-half AB. The line AC is perpendicular to the side opposite point A. CD is one-fourth AC. CE is one-third AC. CF is one-half AC. BD is the hypotenuse of the right triangle BCD.

The Design System

In the Cremonese system the middle of the lower eyes of the violin are placed at the Golden Section of the pin length where a line perpendicular to the centerline intersects the arc AB from point A, which itself is AB from the upper pin. AB equals one-third the pin length of the instrument. The upper eyes are on the arc BD from the lower eyes.

Although there is a simple geometric means of determining an accurate GS of a line, the GS so determined is no longer the geometric sum of the side and the perpendicular of the equilateral triangle. Stradivari therefore would have needed to find a new way to determine a measure comparable to AC, the perpendicular of the equilateral triangle in Figure 1. The simplest alternative effectively reverses the Amati procedure. Instead of finding the GS by geometrically summing the side of the equilateral triangle (AB) and the perpendicular AC, Stradivari could geometrically "subtract" the distance AB from the known GS length, to obtain a measure I refer to as AC*. He would then use AC* to construct an equilateral triangle having AC* as the

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[*] I am grateful to Andrew Dipper, Robert Spear and especially Gary Frisch for questions and suggestions that greatly improved this paper.

[1] Andrew Dipper has noted that Stradivari's first wife was a widow. The father of her first husband was the most eminent Cremonese architect of the 1600s. Dipper cites evidence suggesting that Stradivari himself "may have worked as an architect, or at least worked in a shop or situation in which there were architects." See Andrew Dipper, "The Geometric Construction of the Violin Forms of Antonio Stradivari" VSA Journal Vol. X, No. 2. Perhaps this background explains why Stradivari used a different, more exact, procedure for find the GS. I thank Andrew Dipper for pointing this out to me.

perpendicular. Because AC* is smaller than AC, the subdivisions of AC*, the sides AB*, and DB* would all be a little smaller than on the Amati triangle. The differences are not great, though noticeable even on a violin. They become significant for the larger instruments. For example, for the Davidoff cello the true GS of pin length is 461.6 mm compared to the approximation of 464.6. Using the shorter AC* value to construct the basic triangle one obtains an implied f-hole length about 2 mm shorter than otherwise.

The C-bouts of the violas and cellos are proportionately shorter than for violins, probably reflecting efforts to reduce the stop length to make the instruments more comfortable to play. For these instruments, Strad finds the GS of the pin length as usual. For a violin he would then find AC* as the difference between the GS and one-third the pin length (AB). For the violas and cellos he subtly redefines AC* as the distance between the line passing through the lower eyes and AB. Then he makes several design adjustments that shift the lower eyes upward, thereby shortening AC*. For some instruments he positions the Golden Section length starting from the upper edge rather than the upper pin[2]. He may also, or instead, position the f-holes such that the bottom of the f-hole is on the Golden Section instead of the middle of the lower eyes[3]. Such adjustments to the Davidoff cello shorten AC* by almost 20 mm. Strad's final adjustment to reduce the stop length of the larger instruments and make them more playable is to base the f-hole length on the distance BC* instead of the more usual BD*[4]. The cumulative effect of all these adjustments is quite substantial; they reduce the length of the f-hole on the Servais cello from 141.6 mm (if the f-holes were proportioned and placed like a violin) to only 114.5.

Even though the centers of the lower eyes on the larger instruments do not lie on the GS of the pin length, the lower eyes of all instruments are on the arc AB* from a point on the centerline that is AB below the upper pin. In its fundamental approach, Stradivari's system is unchanged from what Andrea Amati did 150 years earlier.

Operational Guidance for the Workshop

For the Cremonese system as used by the Amatis, Guarneris, and others no distinction is necessary between basic design rules and workshop guidance. Because of the simple relationship between AB, AC, and the Golden Section, the design system is completely operational at the bench. However, Stradivari's system, using AC* rather than AC requires more steps and is more cumbersome, especially for the larger instruments. In a large workshop there would inevitably be errors in application. I believe that Stradivari attempted to avoid these errors by providing simplified diagrams for guidance at the bench. They are, in fact, the well-known and inscrutable diagrams that have been preserved in the Museo Stradivariano in Cremona and reproduced in Sacconi's "Secrets" of Stradivari, Pollen's The Violin Forms of Antonio Stradivari, and elsewhere.

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[2] Tuscan contralto of 1690, 'Medicea' tenor of 1690, Servais cello of 1701, deMunck cello of 1730
[3] 'Medicea' tenor, Servais cello, Davidoff cello of 1712
[4] 'Medicea' tenor, Servais cello, Davidoff cello

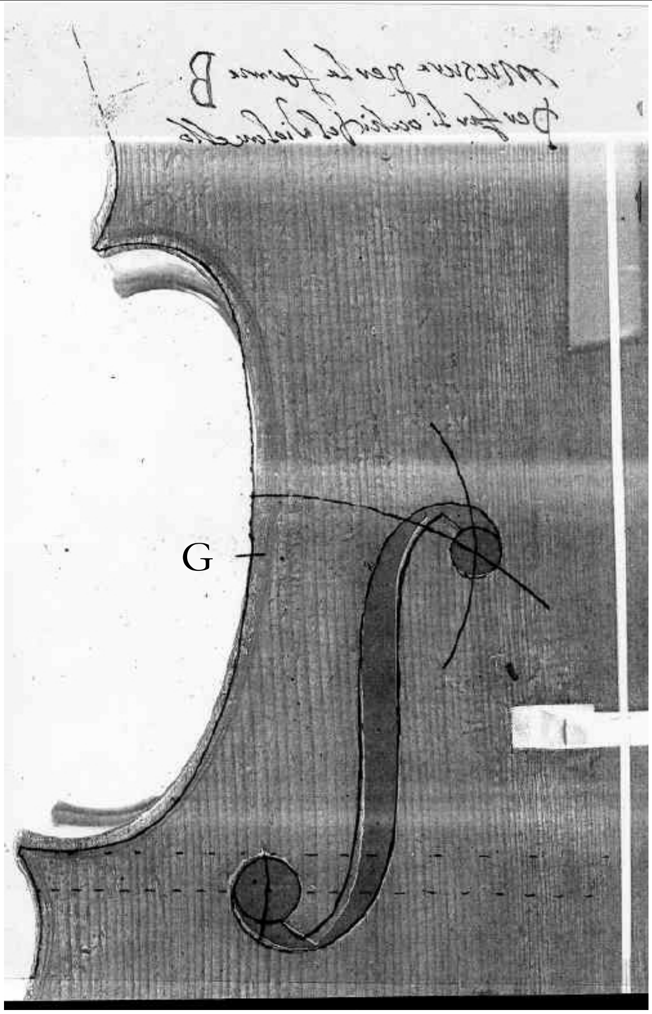

Figure 2 – The Form B f-hole Diagram Overlaid onto the Davidoff Cello

Consider the diagram for the f-holes on the Form B cello, such as the Davidoff of 1712 (Figure 1 of last month's article). Although the instrument "outline" on this diagram looks like a simple tracing or sketch of the rib garland, it is not. Nor is it a tracing or sketch of the edges of the plate. Instead it is a combination of both, carefully drawn for use at the bench. It is easy to verify this by making a photocopy of the diagram and a photocopy of a full size picture of the cello and overlaying them (Figure 2). The "outline" on the diagram matches the plate edges on the upper and lower corners and in the middle of the C-bout. By the extreme lower part of the C-bout, the line has moved gradually inside the edge of the plate to coincide with the scribe line of the rib garland.

On the Form B diagram there are two horizontal lines. The upper touches the "outline" at the low point of the C-bout. The corresponding position on the actual plate is the scribe line of the rib garland. Given this one piece of orientation, the worker could position the f-holes correctly using the arc positions and sizes shown on the diagram. He needed to know nothing about the actual design system.

How was this diagram constructed? With one exception, the sizes and positions of the various arcs used to locate the eyes are all taken directly from the positioning triangle in Figure 1. Triangle dimensions can be inferred from the distance BC* between the upper and lower eyes on the diagram or calculated directly from the pin length of the instrument. The lower horizontal line is placed such that it passes through the centers of the lower eyes. The position in relation to the upper horizontal line would be determined by the actual f-hole design system. In the diagram, the line appears to be lower than the upper line by 1/18 AC*[5]. The center of the lower eye is 1/3 AC* in from the edge of the plate on the lower line, which is why the "outline" at this point must correspond with the actual edge of the plate. The upper eye lies on the scribe line of radius BC* centered on the lower eye. At this point something rather peculiar happens. If Stradivari were simply to open the divider to 1/3 AC*, place one leg of the compass where the arc BC* intersects the plate edge, and mark the position on the arc BC*, he would exactly position the upper eyes. Instead, Stradivari's diagram clearly shows that the compass is to be positioned somewhat below the intersection of the arc BC*

[5] Not difficult to do with a divider: find half of AC*, divide that half into three parts, divide one of these parts into thirds.

and the plate edge [point G in Figure 2]. The position can be located as the length of the f-hole (therefore, ½ AB* or BC*) up from where the line through the lower eyes intersects the plate edge (it is also down by 1/12 AC* from the intersection of the arc through the upper eyes and the plate edge). The divider is then set to a width something less than 1/3 AC* and the position of the upper eye is marked.[6]

The role of point G is setting the position of the upper eye is the aspect of Stradivari's system that is least clear to me. I suspect that point G highlights the difference between the design system and the workshop practice. At the design stage, point G is unnecessary, as Stradivari knew AC* and the various subdivisions that actually position the eyes. At the bench, point G is useful because the worker does not know AC* or the subdivisions. It is easy to set point G and easy to use the divider and diagram to set the arc that fixes the upper eye.

To demonstrate that the diagram is constructed as I have described, consider the numbers for the Form B diagram, using the image from Pollens' book. The distance between the upper and lower eyes I measure at 114.7 mm. On a cello this is BC*, or half the side of the basic triangle. All calculated distances in Table 1 are derived from this length, using the equations given previously.

Table 1

	f-hole length (BC*)	Height of triangle (AC*)	1/3 AC*	1/12 AC*	1/18 AC*
Calculated lengths	NA	198.7	66.2	16.6	11.0
Measured lengths	114.7	NA	66.0	16.7	11.5

The bottom of the f-hole is 17 mm (1/12 AC*) below the line through the center of the lower eye, another example of how Stradivari proportioned the instrument in relation to the triangle.

There are similar diagrams for the piccolo cello, the tenor and contralto violas, and for different models of the violin. The relationship of the diagrams to the underlying design schemes is usually as I have described for the Form B cello. Only for the contralto and tenor violas of 1690 does the diagram "outline" correspond to the rib garland. Perhaps Stradivari found that this gave imprecise guidance, especially in a large workshop, and made changes. The guidance for setting the lower eyes on the violins varied. For example, on the P and G models, the eyes are located using arcs centered on the lower inside portion of the C-bout edge. In these cases, the centers of the lower eyes are 1/6 AC* in from the edge of the plate. The positioning arc is placed exactly at the point where the "outline" on the diagram moves away from the plate edge and begins to move in toward the rib garland. This provides unambiguous guidance to the workman.

Some Examples

The Betts of 1704

The pin length of the Betts is violin of 1704 is 345 mm, giving AB a value of 115 mm. The Golden Section of the pin length is 213.2 mm. Subtracting AB from the Golden Section gives AC* of 98.2 mm (=213.2 – 115). From this, we can calculate the length of the f-hole for the Betts as 61.8 mm.

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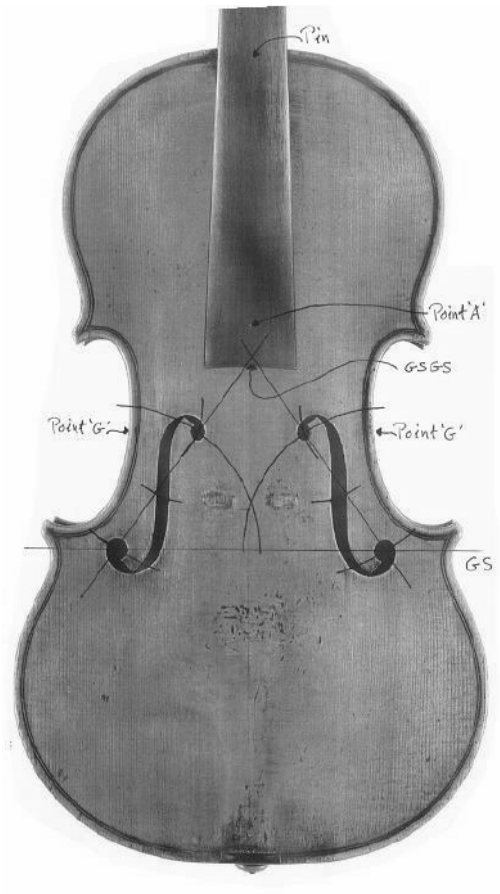
[6] The distance from that point to the middle of the upper eye is 64 mm, which corresponds to nothing from the basic triangle or AC* (although it is 2/3 of the distance between the lower eye and point G).

This equals the actual length of 61.8 mm. The lower eyes are at the intersection of the GS and arc AB* from point A on the centerline. The upper eyes are on the arc BD* centered on the lower eye, 1/3AC* in from the plate edge. The notches on the f-holes are at 1/3AC* measuring from the middle of the arc BD* across the upper wing and from the comparable spot on lower wing.

The lower eyes on the Betts are closer together than on Stradivari's diagram for the P form. This gives the soundholes a somewhat upright appearance, a point that Hargrave has noted (*The Strad* May 1989). A consequence is that, unlike many Stradivari instruments, the lines through the upper and lower eyes cross above, not at, the GSGS.

Figure 3 'Betts' Stradivari showing placement of f-holes



Davidoff cello of 1712 (Form B)

The pin length of the Davidoff is 747 mm, giving a Golden Section of 461.6 mm. The bottom of the f-holes are positioned to touch this line. The middle of the lower eyes are about 16.5 mm higher (1/12 AC*). One third the pin length is 249 mm. AC* is therefore 196.1 mm (=461.6 - 16.5 - 249). From this, we can calculate AB* and then the length of the f-hole for the Davidoff as 113.2 mm. This compares quite closely to the actual length of 114. The lower eyes are on the arc AB* from point A. As a matter of design, the upper eyes can be positioned in multiple ways. They are on the arc BC* centered on the lower eye, 1/3AC* in from the plate edge. As with many violins, the lines through the eyes cross at the Golden Section of the distance between the middle of the eyes and the upper pin, in this case about 275 mm below the upper pin. Hence, the centers of the upper eyes are at the intersection of arc BC* and the line from the lower eyes to the GSGS. They are also BC* down from point A. At the bench, the eyes were probably placed from point G according to the diagram.

Figure 4 – 'Davidoff' Stradivari cello showing placement of f-holes

Results in Table Form

In the table below I summarize the results for various instruments by Stradivari. Note that for instruments after 1690, I find the implied f-hole length using the true value for GS to derive AC*. Using the value appropriate to Amati yields somewhat inferior results.

Table 2					
Instrument	Body Length	Pin Length	1/3 Pin Length	Implied f-hole Length	Measured f-hole Length
'Tullaye' violin 1670	352	342.4	114.1	62.2	61.7
'Harrison' violin 1693	362	352.8	117.6	63.2	62.8
'Betts' violin 1704	355.4	345	115	61.8	61.8
'Soil' violin 1714	356.5	345.9	115.3	62.0	62.4
'Cremonese' violin 1715	355.5	345.5	115.2	61.9	61.8
'Milanollo' violin 1728 Forma PG	354	344.5	114.8	61.7	63 [a] 62 [b]

'Prince Khevenhueller' violin 1733	358	347	115.7	62.1	62.5 [a] 61.8 [b]
Tuscan contralto viola 1690 [e]	412	400.4	133.5	69.0	68[a] 69 [b]
Tuscan tenor viola 'Medicea' 1690	475	459	153.0	72.2	72
'Servais' cello 1701	792	779.4	258.8	114.4	114.5
'Davidoff' cello 1712	758	747	249	113.2	114
Notes to Table: [a] treble; [b] bass					

The Mystery is Solved—Long Live the Mystery!

Stradivari made only one significant change to the usual Cremonese system for determining the size and position of the f-holes: he changed the measure of the Golden Section. However, that one change complicated bench procedures. I believe that Stradivari created his diagrams to help at the bench and that the reason no similar diagrams exist for other makers is that they were unnecessary.

Using the system I have set out, a maker today can create his or her own personal design while retaining the look of the old Italian instruments. But there is still a mystery to be solved. Why does the distance between the pins play a critical role? Some makers place the pins well inside the outer edge; some place them right at the purfling. No matter where the pins are, that distance determines soundhole layout. I look forward to the solution to this new mystery.